Observational Techniques

Exercise No. 2: Photometric Standard Calibration with Linear Least Squares

Due date: Dec 28, 2008 (last candle of Chanuka)

A star has a Vega-based R-band magnitude R, and a color $V - I$ (defined as its V magnitude minus its R magnitude; if $V - I = 0$ its color is the same as that of Vega, if it's negative it is bluer than Vega, if it's positive it is redder than Vega). The star is observed through an R filter at a zenith angle z, i.e. at an airmass of $AM = 1/\cos(z)$, with an exposure time t, and produces N counts, i.e., a count rate of N/t. The gain of the detector is g electrons/ADU. The star is much brighter than the sky, so the error in its counts are photon-noise dominated.

If the detector is linear and the atmospheric conditions are "photometric" (i.e. if one can perform photometry to few-percent accuracy, which will be tested below) then we expect these variables to obey:

$$R = -2.5\log(N/t) + R0 - \text{extinct} \times AM - \text{color} \times (V - I),$$

where $R0$, "extinct", and "color" are parameters to be determined for the particular observational setup and the atmospheric conditions on the observing night. $R0$ is the photometric zeropoint, i.e., the R magnitude of a star of $V - I = 0$ that would give, above the atmosphere (AM=0), 1 count per second. "extinct" is the extinction coefficient, in units of magnitudes per unit air mass. "color" is the color term coefficient, that accounts for the possibility that the system's filter+detector wavelength response is different from the one originally used to define the magnitude of the star. (We could equally well choose some other color instead of $(V - I)$, e.g., $B - V$ or $V - R$ to calibrate this effect.)

The ascii-format file `standardphotometry.asc`, also on this website, provides actual measurements of a set of standard stars at various airmasses during a certain night. There are three rows for every measurement. The first three numbers in the first row are the exposure time t [s], AM, and g [e/ADU] (ignore the rest of that row). The first three numbers in the second row are the central x, y positions of the star on the detector (you don't care about them) and the N counts measured within the seeing disk. (Again, ignore the rest of this row). The third line gives the previously known magnitudes and colors of each star, as calibrated by A. Landolt (1992, *Astronomical Journal*, 104, 340; this is the standard work for optical photometric calibration). The entries are $V,\ B-V,\ U-B,\ V-R,\ R-I,\ V-I$. Thus to get the previously calibrated R magnitude of each star, you need to do $V - (V - R)$ using the first and fourth entry, and you can take the $V - I$ color from the 6th entry.

a. Define new parameters such that the above equation assumes a simple linear form,

$$y = a_1 x_1 + a_2 x_2 + a_3 x_3,$$

e.g., $y \equiv R + 2.5 \log N/t$, $a_1 \equiv R0$, $x_1 \equiv 1$, etc. Then, for the set of $i = 1, n$ measurements, write an expression for the covariance matrix of the independent variables, x_1, x_2, x_3, using the errors σ_i in the observables, y_i. You will need to calculate the σ_i's based on the counts N and the gain g, propagating the errors correctly.
b. Write a computer program that solves for the inverse of the covariance matrix (i.e., the covariance matrix of the parameters a_j) given the data, and then finds the best-fitting parameters $R0$, "extinct", "color", and their uncertainties. You may use library routines for matrix inversion, and vector and matrix multiplication, but must write the rest of the program yourself, in any lower or upper-level language you prefer. (In other words, you may NOT just feed the numbers into an existing program that, e.g., "calculates a linear least squares solution to input data vectors", as you would learn little from doing this.)
c. Calculate the χ^2 of the best-fit solution, and find the amount of "cosmic variance" σ_c^2, that must be added to the measurement errors in order to make χ^2 approximately equal to the degrees of freedom. Remember that each guess of cosmic variance changes the original covariance matrix, so you must solve the problem repeatedly until you get the "right" cosmic variance.
d. A shortcoming of maximum likelihood methods is their sensitivity to "outlier" data, which often exist in real life, due to systematic errors or corrupted data. Check for the existence of such outliers in the present dataset. If you find any, repeat the fit without them, to obtain your final solution.

Please submit only the following:
1. A printout of your source code (with some comments, so I can understand what you're doing).
2. Your final best-fitting values for $R0$, "extinct", "color", their uncertainties, and $\sigma_{\rm cosmic}$.
3. The 3×3 covariance matrix of the parameters.
4. A two-column list of R(Landolt) [i.e. $V - (V - R)$ directly from the file above], and R(observed) with its errors. R(observed) is the R you calculate for each star with the best fit parameters you found. The errors should be calculated accounting for the covariances among the parameters, plus the cosmic scatter.
5. A plot showing the data from item 4., i.e., R(observed) vs. R(Landolt) with the calculated data points and their error bars, and the ideal line, R(observed) = R(Landolt).

Note: The data provided here *are* "photometric", in the sense that they can be fit with the above relation with a scatter of a few percent. If this is not what you find, you are doing something wrong!