Now consider the case of isotropic pressure. To calculate the pressure, consider the force dF in the z direction of the particles on a piston (or the side of a box containing the particles) of small area dA that is perpendicular to the z direction (note: in this subsection z denotes a Cartesian coordinate, not redshift). The pressure is defined as

$$p = \frac{dF}{dA} = \frac{dq_z/dt}{dA} \ ,$$

where dq_z is the z component of momentum imparted to the piston, and we used the z component of Newton's law in the form that is also valid in special relativity. Particles with z momentum q_z contribute $2q_z$ per particle to dq_z (with the factor of 2 due to elastic recoil). Particles with z velocity v_z can reach the piston during the time dt from as far away as $dz = v_z dt$. Thus,

$$dq_z = \int 2q_z \, f \, d^3q \, dA \, v_z \, dt \ .$$

Now, in general (in special relativity) $\vec{q} = \gamma m \vec{v}$, where m is the particle mass and $\gamma = 1/\sqrt{1-v^2}$. Also, isotropy implies:

$$\langle v_x^2 \rangle = \langle v_y^2 \rangle = \langle v_z^2 \rangle = \frac{1}{3} \langle v^2 \rangle \ ,$$

where the averaging here is over all the particles at a given position. Thus, $\langle q_z \, v_z \rangle = \frac{1}{3} \langle q \, v \rangle$, but we must then add a factor of 1/2 since in this averaging we must count only the half of particles headed towards the piston (rather than the opposite z direction). Thus,

$$dq_z = \frac{1}{3} \int q \, v \, f \, d^3q \, dA \, dt \ ,$$

so that

$$p = \frac{1}{3} \int q \, v \, f \, d^3q \ . \tag{2.29}$$