Galaxies

Lecture 2

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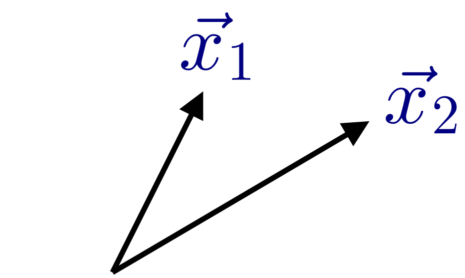
http://wise-obs.tau.ac.il/~barkana/galaxies.html

Statistics

Ergodic Hypothesis

2-point correlation function

Continuous (Field)

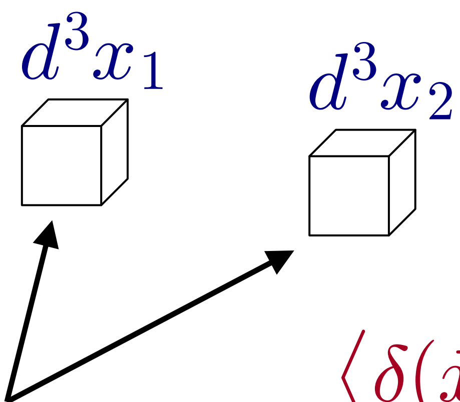


$$\xi(r_{12}) = \left\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \right\rangle$$

$$r_{12} = \left| \vec{x}_1 - \vec{x}_2 \right|, \; \left\langle \delta(\vec{x}) \right\rangle = 0$$

Discrete (Point Process)

Pair distribution function



$$dP(\vec{x}_1, \vec{x}_2) = \bar{n}^2 d^3x_1 d^3x_2 [1 + \xi(r_{12})]$$

$$\left\langle \delta(\vec{x}_1)\delta(\vec{x}_2) \right\rangle^d = \xi(r_{12}) + \bar{n}^{-1}\delta^D(\vec{x}_1 - \vec{x}_2)$$

Statistics

Discrete (Point Process) 3-point correlation function

$$dP(1,2,3) = \prod_{k=1}^{3} (\bar{n} d^3 x_k) \left[1 + \xi_{12}^{(2)} + \xi_{23}^{(2)} + \xi_{31}^{(2)} + \xi_{123}^{(3)}\right]$$

Continuous (Field) 3-point correlation function

$$\langle \delta_1 \delta_2 \delta_3 \rangle = \langle \delta_1 \rangle_c \langle \delta_2 \rangle_c \langle \delta_3 \rangle_c + \langle \delta_1 \delta_2 \rangle_c \langle \delta_3 \rangle_c$$

Cluster expansion

Cumulants

Irreducible moments

$$+ \langle \delta_2 \delta_3 \rangle_c \langle \delta_1 \rangle_c + \langle \delta_3 \delta_1 \rangle_c \langle \delta_2 \rangle_c$$

$$+ \langle \delta_1 \delta_2 \delta_3 \rangle_c$$

Power Spectrum

$$\hat{\delta}(\vec{k}) \equiv \int \frac{d^3x}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}}\delta(\vec{x})\,; \quad \delta(\vec{x}) = \int d^3k\, e^{i\vec{k}\cdot\vec{x}}\,\hat{\delta}(\vec{k})$$

$$\delta^{\mathrm{D}}(\vec{k}) = \int \frac{d^3x}{(2\pi)^3}\, e^{\pm i\vec{k}\cdot\vec{x}}$$

$$\left\langle \hat{\delta}(\vec{k}_1)\hat{\delta}(\vec{k}_2)\right\rangle \equiv P(k_1)\delta^{\mathrm{D}}(\vec{k}_1+\vec{k}_2)$$

Power Spectral Density

$$P(k) = \int \frac{d^3x}{(2\pi)^3}\, e^{-i\vec{k}\cdot\vec{x}}\,\xi(r)\,; \quad \xi(r) = \int d^3k\, e^{i\vec{k}\cdot\vec{x}} P(k)$$

Wiener-Khintchine theorem

Discrete Power Spectrum

$$\triangle x = L/n\,; \;\; \triangle k = 2\pi/L$$

Periodic Cube

$$\vec{x}_{\vec{j}} = (j_x, j_y, j_z)\triangle x, \;\; 0 \le j_x, j_y, j_z < n$$

$$\vec{k}_{\vec{l}} = (l_x, l_y, l_z)\triangle k, \;\; 0 \le l_x, l_y, l_z < n$$

$$\hat{\delta}(\vec{k}_{\vec{l}}) \equiv n^{-3} \sum_{\vec{j}} e^{-i\vec{k}_{\vec{l}}\cdot\vec{x}_{\vec{j}}} \delta(\vec{x}_{\vec{j}})\,;$$

$$\delta(\vec{x}_{\vec{j}}) = \sum_{\vec{l}} e^{i\vec{k}_{\vec{l}}\cdot\vec{x}_{\vec{j}}} \hat{\delta}(\vec{k}_{\vec{l}})$$

Gaussian Random Field

$P(k) \Leftrightarrow \xi(r)$ Fourier transform pairs

Meaning of discrete P(k): $\left(\mathrm{Re}\,\hat{\delta}(\vec{k}_{\vec{l}}), \mathrm{Im}\,\hat{\delta}(\vec{k}_{\vec{l}})\right)$:

$$\mathrm{Ave} = 0, \quad \mathrm{Var} = \tfrac{1}{2} P(k)\,(\triangle k)^3$$

Alternatively, define: $\hat{\delta}(\vec{k}_{\vec{l}}) = |\hat{\delta}_{\vec{k}}| e^{i\phi_{\vec{k}}}$

$$0 \leq \phi_{\vec{k}} < 2\pi \;\; (\mathrm{Random\;\; Phases})$$

$$|\delta_{\vec{k}}| \,:\; p(x)dx = \sigma^{-2} e^{-x^2/2\sigma^2} x\,dx$$

$$\sigma^2 = P(k)\,(\triangle k)^3$$

(Rayleigh Distribution)