Galaxies

Lecture 9

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http://wise-obs.tau.ac.il/~barkana/galaxies.html

General Distribution Function

$$dN = f(\vec{x}, \vec{q}, t)\, d^3x\, d^3q \qquad q = mv$$

$$n(\vec{x}, t) = \int f\, d^3q \qquad \rho = mn$$

$$\bar{q}_i = \frac{1}{n} \int q_i\, f\, d^3q \qquad \tilde{q}_i = q_i - \bar{q}_i$$

$$T_{ij} = \int \frac{\tilde{q}_i \tilde{q}_j}{m} f\, d^3q$$

Stress Tensor

$$\pi_{ij} = \int \frac{3\tilde{q}_i \tilde{q}_j - \tilde{q}^2 \delta_{ij}}{3m} f\, d^3q$$

Anisotropic Stress Tensor

Stellar Distribution Function

$$dN = f(\vec{x}, \vec{v}, t) d^3x d^3v$$

$$n(\vec{x}, t) = \int f\, d^3v$$

$$\bar{v}_i = \frac{1}{n} \int v_i\, f\, d^3v \qquad \tilde{v}_i = v_i - \bar{v}_i$$

$$\sigma^2_{ij} = \frac{1}{n} \int \tilde{v}_i \tilde{v}_j\, f\, d^3v$$

Velocity Tensor

Liouville Equation

Collisionless Boltzmann Equation

$$\frac{df}{dt} = \left[\frac{\partial}{\partial t} + \frac{d\vec{x}}{dt}\cdot\frac{\partial}{\partial\vec{x}} + \frac{d\vec{q}}{dt}\cdot\frac{\partial}{\partial\vec{q}}\right] f = 0$$

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{u}) = 0$$

Continuity Equation (mass conservation)

$$\frac{\partial\vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}\phi - \frac{1}{\rho}\vec{\nabla}p - \frac{1}{\rho}\vec{\nabla}\cdot\pi$$

Euler Equation (momentum conservation)

$$\nabla^2\phi = 4\pi G\rho$$

Poisson Equation

Liouville Equation

Collisionless Boltzmann Equation: for stars

$$\frac{df}{dt} = \Big[\frac{\partial}{\partial t} + \frac{d\vec{x}}{dt}\cdot\frac{\partial}{\partial \vec{x}} + \frac{d\vec{v}}{dt}\cdot\frac{\partial}{\partial \vec{v}}\Big] f = 0$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x_i}(n\bar{v}_i) = 0$$

Continuity Equation (number conservation)

$$\frac{\partial \bar{v}_j}{\partial t} + \bar{v}_i \frac{\partial}{\partial x_i}\bar{v}_j = -\frac{\partial}{\partial x_j}\Phi - \frac{1}{n}\frac{\partial}{\partial x_i}(n\sigma^2_{ij})$$

"Euler" Equation (momentum conservation)

Jeans Equations

$$\nabla^2\Phi = 4\pi G\rho$$

Poisson Equation

Spherical Coordinates

Acceleration
(Binney & Tremaine 1B-31)

$$\frac{d\vec{v}}{dt} = (\ddot{R} - R\dot{\theta}^2 - R\sin^2\theta\dot{\phi}^2)\hat{R}$$
$$+ (2\dot{R}\dot{\theta} + R\ddot{\theta} - R\sin\theta\cos\theta\dot{\phi}^2)\hat{\theta}$$
$$+ (R\sin\theta\ddot{\phi} + 2\dot{R}\sin\theta\dot{\phi} + 2R\cos\theta\dot{\phi}\dot{\theta})\hat{\phi}$$

Collisionless Boltzmann Equation
(Binney & Tremaine 4P-2)

$$0 = \frac{\partial f}{\partial t} + v_R\frac{\partial f}{\partial R} + \frac{v_\theta}{R}\frac{\partial f}{\partial \theta} + \frac{v_\phi}{R\sin\theta}\frac{\partial f}{\partial \phi} + \left(\frac{v_\theta^2+v_\phi^2}{R} - \frac{\partial \Phi}{\partial R}\right)\frac{\partial f}{\partial v_R}$$
$$+ \frac{1}{R}\left(v_\phi^2\cot\theta - v_R v_\theta - \frac{\partial \Phi}{\partial \theta}\right)\frac{\partial f}{\partial v_\theta}$$
$$- \frac{1}{R}\left[v_\phi(v_R + v_\theta\cot\theta) + \frac{1}{\sin\theta}\frac{\partial \Phi}{\partial \phi}\right]\frac{\partial f}{\partial v_\phi}$$

Apply to CBE: $\int v_R \, d^3v$:

Assume Steady State: $\frac{\partial f}{\partial t} = 0$

also need: $\overline{v_R \cdot v_\theta} = 0$

Spherically Symmetric: $\frac{\partial f}{\partial \theta} = \frac{\partial f}{\partial \phi} = 0$

$$\frac{d\left(n\overline{v_R^2}\right)}{dR} + \frac{n}{R}\left[2\overline{v_R^2} - \left(\overline{v_\theta^2} + \overline{v_\phi^2}\right)\right] = -\, n\frac{d\Phi}{dR}$$

$$\ldots\ldots \quad \frac{1}{n}\frac{d}{dR}\left(n\overline{v_R^2}\right) + 2\beta\frac{\overline{v_R^2}}{R} = -\frac{d\Phi}{dR}$$

Functions: $n(R), \ \overline{v_R^2}(R), \ \beta(R) => \Phi(R) => M(R)$

Measure: $I(R_p) => n, \ \sigma^2(R_p) => \overline{v_R^2}$