Galaxies

Lecture 11

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http://wise-obs.tau.ac.il/~barkana/galaxies.html

Euler equation: Gas disk $(z = 0)$
(Include gravity and pressure)

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \vec{\nabla})\vec{v} = -\frac{1}{\rho}\vec{\nabla} p - \vec{\nabla}\Phi$$

In cylindrical coordinates:

$$\frac{\partial v_R}{\partial t} + v_R \frac{\partial v_R}{\partial R} + \frac{v_\phi}{R}\frac{\partial v_R}{\partial \phi} - \frac{v_\phi^2}{R} = -\frac{\partial \Phi}{\partial R} - \frac{1}{\Sigma}\frac{\partial p}{\partial R}$$

$$\frac{\partial v_\phi}{\partial t} + v_R \frac{\partial v_\phi}{\partial R} + \frac{v_\phi}{R}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_R}{R} = -\frac{1}{R}\frac{\partial \Phi}{\partial \phi} - \frac{1}{\Sigma R}\frac{\partial p}{\partial \phi}$$

Now assume a simple equation of state: $p = K\Sigma^\gamma$

The speed of sound: $c_s^2 = \frac{dp}{d\Sigma} = \gamma K \Sigma^{\gamma - 1}$

Now simplify using specific enthalpy h:

$$\frac{\partial v_R}{\partial t} + v_R \frac{\partial v_R}{\partial R} + \frac{v_\phi}{R}\frac{\partial v_R}{\partial \phi} - \frac{v_\phi^2}{R} = -\frac{\partial}{\partial R}(\Phi + h)$$

$$\frac{\partial v_\phi}{\partial t} + v_R \frac{\partial v_\phi}{\partial R} + \frac{v_\phi}{R}\frac{\partial v_\phi}{\partial \phi} + \frac{v_\phi v_R}{R} = -\frac{1}{R}\frac{\partial}{\partial \phi}(\Phi + h)$$

Unperturbed equation: $\frac{v_{\phi,0}^2}{R} = \frac{d}{dR}(\Phi_0 + h_0)$

Perturbed variables:

$v_R = v_{R,1}$ $\qquad$ $v_\phi = v_{\phi,0} + v_{\phi,1}$

$h = h_0 + h_1$ $\qquad$ $\Phi = \Phi_0 + \Phi_1$

1'st order equations (subtracted 0'th order):

$$\frac{\partial v_{R,1}}{\partial t} + \Omega \frac{\partial v_{R,1}}{\partial \phi} - 2\Omega v_{\phi,1} = -\frac{\partial}{\partial R}(\Phi_1 + h_1)$$

$$\frac{\partial v_{\phi,1}}{\partial t} + \left[\frac{d(\Omega R)}{dR} + \Omega\right] v_{R,1} + \Omega \frac{\partial v_{\phi,1}}{\partial \phi} = -\frac{1}{R}\frac{\partial}{\partial \phi}(\Phi_1 + h_1)$$

$$\frac{\partial v_{R,1}}{\partial t} + \Omega \frac{\partial v_{R,1}}{\partial \phi} - 2\Omega v_{\phi,1} = -\frac{\partial}{\partial R}(\Phi_1 + h_1)$$

$$\frac{\partial v_{\phi,1}}{\partial t} + \left[\frac{d(\Omega R)}{dR} + \Omega\right] v_{R,1} + \Omega \frac{\partial v_{\phi,1}}{\partial \phi} = -\frac{1}{R}\frac{\partial}{\partial \phi}(\Phi_1 + h_1)$$

Search for wave solutions:

$$v_{R,1} = \mathrm{Re}\left[\, v_{R,a}(R) e^{i(m\phi - \omega t)}\right]$$

$$v_{\phi,1}\,;\ \Phi_1\,;\ h_1\,;\ \Sigma_1$$

- • • Substitute; Solve for velocity components =>

$$v_{R,a} = -\frac{i}{\triangle}\left[(m\Omega - \omega)\frac{d}{dR}(\Phi_a + h_a) + \frac{2m\Omega}{R}(\Phi_a + h_a)\right]$$

$$v_{\phi,a} = \frac{1}{\triangle}\left[-2B\frac{d}{dR}(\Phi_a + h_a) + \frac{m(m\Omega - \omega)}{R}(\Phi_a + h_a)\right]$$

$$v_{R,a} = -\frac{i}{\triangle}\left[(m\Omega-\omega)\frac{d}{dR}(\Phi_a+h_a)+\frac{2m\Omega}{R}(\Phi_a+h_a)\right]$$

$$v_{\phi,a} = \frac{1}{\triangle}\left[-2B\frac{d}{dR}(\Phi_a+h_a)+\frac{m(m\Omega-\omega)}{R}(\Phi_a+h_a)\right]$$

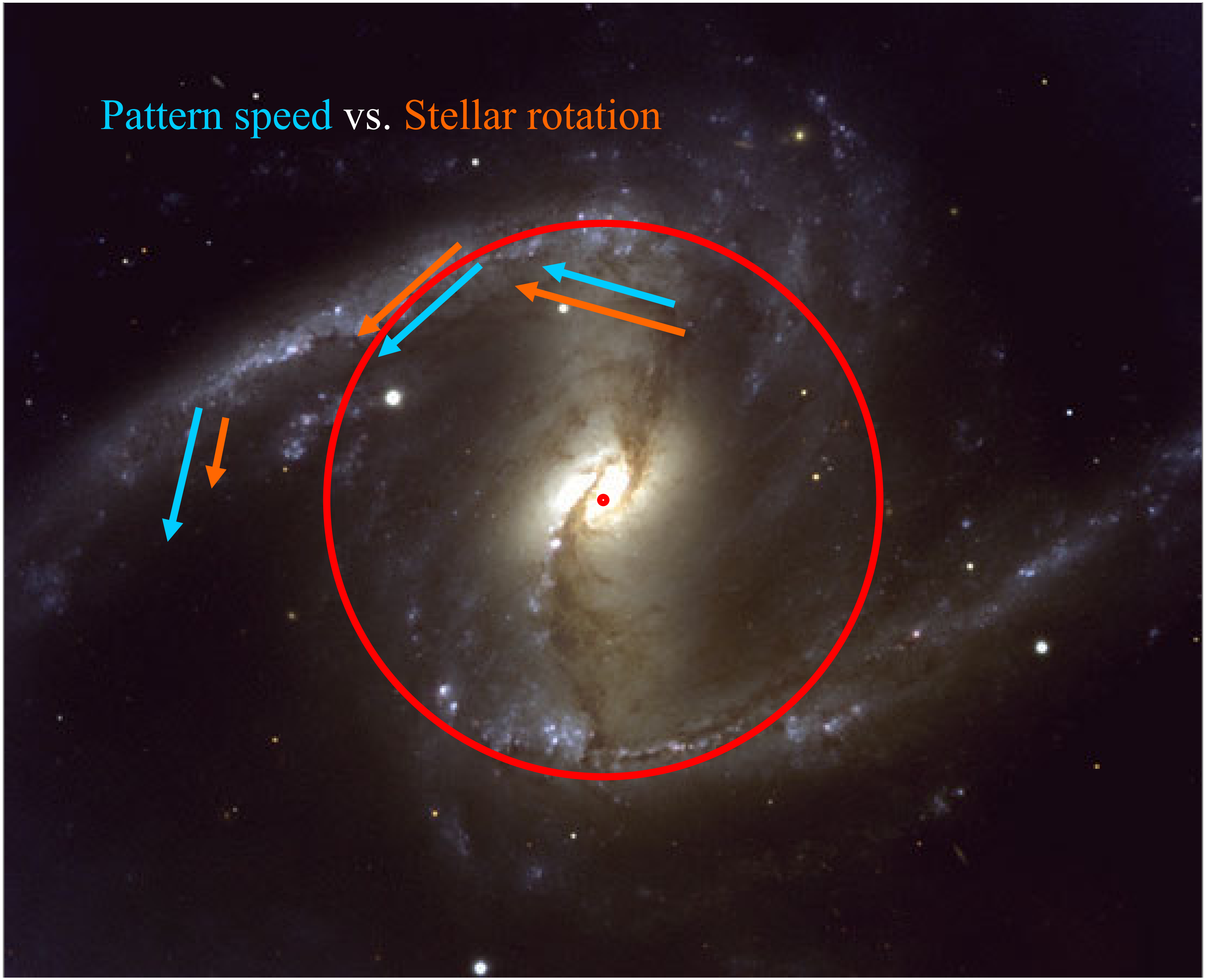
$$\triangle \equiv \kappa^2-(m\Omega-\omega)^2$$

1’st order eq. of state:

$$h_a = c_s^2(\Sigma_0)\frac{\Sigma_a}{\Sigma_0}$$

Pattern speed: $\Omega_p = \frac{\omega}{m}$

Exponent of wave: $i(m\phi-\omega t) = im(\phi-\Omega_p t)$


Pattern speed vs. Stellar rotation

Continuity equation: Gas disk

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0$$

In cylindrical coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{R}\frac{\partial (R\rho v_R)}{\partial R} + \frac{1}{R}\frac{\partial (\rho v_\phi)}{\partial \phi} + \frac{\partial (\rho v_z)}{\partial z} = 0$$

For a disk, 1'st order:

$$\frac{\partial \Sigma_1}{\partial t} + \frac{1}{R}\frac{\partial}{\partial R}(R\Sigma_0 v_{R,1}) + \Omega \frac{\partial \Sigma_1}{\partial \phi} + \frac{\Sigma_0}{R}\frac{\partial v_{\phi,1}}{\partial \phi} = 0$$

For wave solution:

$$i(m\Omega - \omega)\Sigma_a + \frac{1}{R}\frac{d}{dR}(R\Sigma_0 v_{R,a}) + \frac{im\Sigma_0}{R} v_{\phi,a} = 0$$

Summary: 5 equations, 5 unknowns

Euler equation

$$v_{R,a} = -\frac{i}{\triangle}\left[(m\Omega - \omega)\frac{d}{dR}(\Phi_a + h_a) + \frac{2m\Omega}{R}(\Phi_a + h_a)\right]$$

$$v_{\phi,a} = \frac{1}{\triangle}\left[-2B\frac{d}{dR}(\Phi_a + h_a) + \frac{m(m\Omega-\omega)}{R}(\Phi_a + h_a)\right]$$

Continuity equation

$$i(m\Omega - \omega)\Sigma_a + \frac{1}{R}\frac{d}{dR}(R\Sigma_0 v_{R,a}) + \frac{im\Sigma_0}{R}v_{\phi,a} = 0$$

Equation of state

$$h_a = c_s^2(\Sigma_0)\frac{\Sigma_a}{\Sigma_0}$$

+ Poisson Equation

Tight-winding approximation:

$$\Sigma_1(R,\phi,t) = H(R,t)e^{i[m\phi+f(R,t)]}$$

$$\Phi_1(R,\phi,t) = -\frac{2\pi G}{|k|}\Sigma_1(R,\phi,t)$$ (Used Poisson Eq.)

$$k(R,t) = \frac{\partial f(R,t)}{\partial R} \qquad \cot i = \left|\frac{kR}{m}\right| \gg 1$$

Apply it here:

$$\Sigma_a = H(R)e^{if(R)} \qquad k = \frac{df}{dR}$$

$$\Phi_a = -\frac{2\pi G}{|k|}H(R)e^{if(R)}$$

$$h_a = G(R)e^{if(R)}$$ (H, G are smooth, f varies rapidly)

Euler equation(s)

$$v_{R,a} = -\frac{i}{\triangle}\left[(m\Omega - \omega)\frac{d}{dR}(\Phi_a + h_a) + \frac{2m\Omega}{R}(\Phi_a + h_a)\right]$$

$$v_{R,a} = \frac{1}{\triangle}(m\Omega - \omega)k(\Phi_a + h_a)$$

$$v_{\phi,a} = \frac{1}{\triangle}\left[-2B\frac{d}{dR}(\Phi_a + h_a) + \frac{m(m\Omega-\omega)}{R}(\Phi_a + h_a)\right]$$

$$v_{\phi,a} = -\frac{2i}{\triangle}Bk(\Phi_a + h_a)$$

Continuity equation

$$i(m\Omega - \omega)\Sigma_a + \frac{1}{R}\frac{d}{dR}(R\Sigma_0 v_{R,a}) + \frac{im\Sigma_0}{R}v_{\phi,a} = 0$$

$$(m\Omega - \omega)\Sigma_a + k\Sigma_0 v_{R,a} = 0$$ [Dropped 1/(kR) last term]

5 equations:

$$\begin{cases} v_{R,a} = \frac{1}{\triangle}(m\Omega - \omega)k(\Phi_a + h_a) \\ v_{\phi,a} = -\frac{2i}{\triangle}Bk(\Phi_a + h_a) \\ (m\Omega - \omega)\Sigma_a + k\Sigma_0 v_{R,a} = 0 \\ h_a = c_s^2(\Sigma_0)\frac{\Sigma_a}{\Sigma_0} \\ \Phi_a = -\frac{2\pi G}{|k|}\Sigma_a \end{cases}$$

(Continuity equation)

$$=> \quad \Sigma_a\left(1 + \frac{k^2c_s^2}{\triangle} - \frac{2\pi G\Sigma_0|k|}{\triangle}\right) = 0$$

$$\triangle \equiv \kappa^2 - (m\Omega - \omega)^2$$

$$=> \quad (m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2c_s^2$$

Dispersion relation

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 c_s^2$$ Gas disk

$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k|\, \mathcal{F}\left(\frac{\omega - m\Omega}{\kappa}, \frac{k^2\sigma_R^2}{\kappa^2}\right)$$

Stellar disk (B&T 6-46)

We have established local solutions √

But not a full global solution:

1) Assumed tight winding
2) Boundary conditions (Disk center & edge)
3) Singularities (Resonances)

Basic reason for spiral structure:
Minimize energy while conserving angular momentum

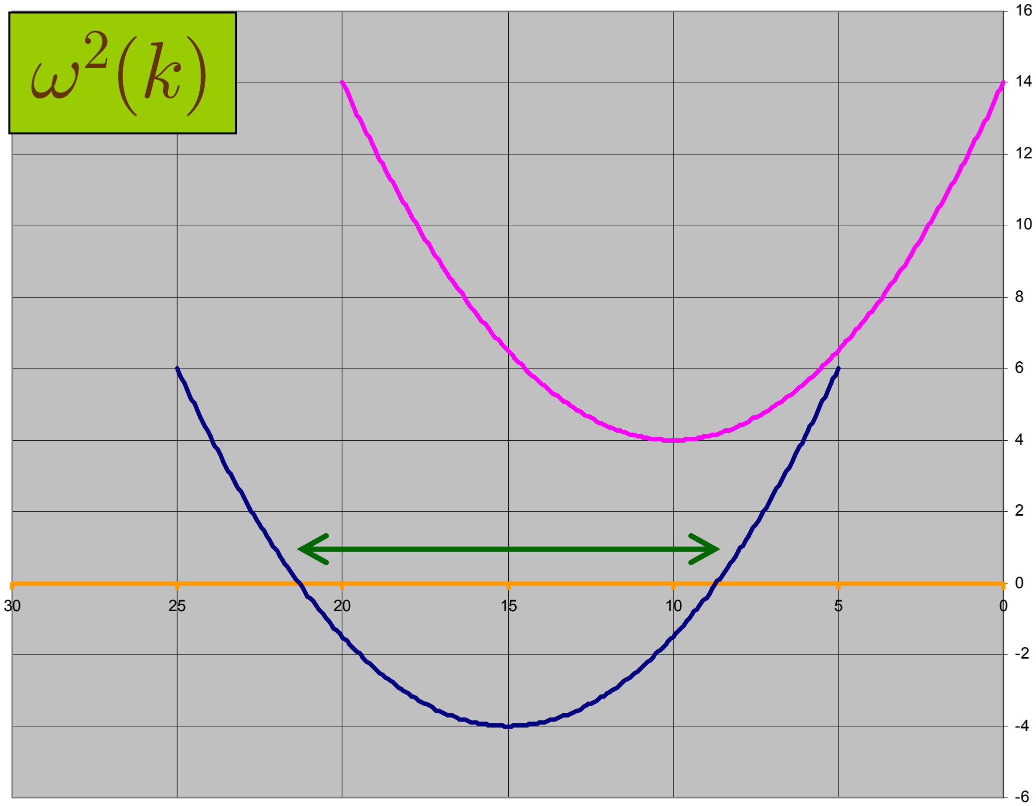
$$(m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 c_s^2$$ Gas disk

Consider now axisymmetric perturbations, m=0:

Same equation as long as: $|kR| \gg 1$

$$\omega^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 c_s^2$$

$$\Sigma \propto e^{-i\omega t}$$



$$Q \equiv \frac{c_s \kappa}{\pi G\Sigma} > 1$$

Stability

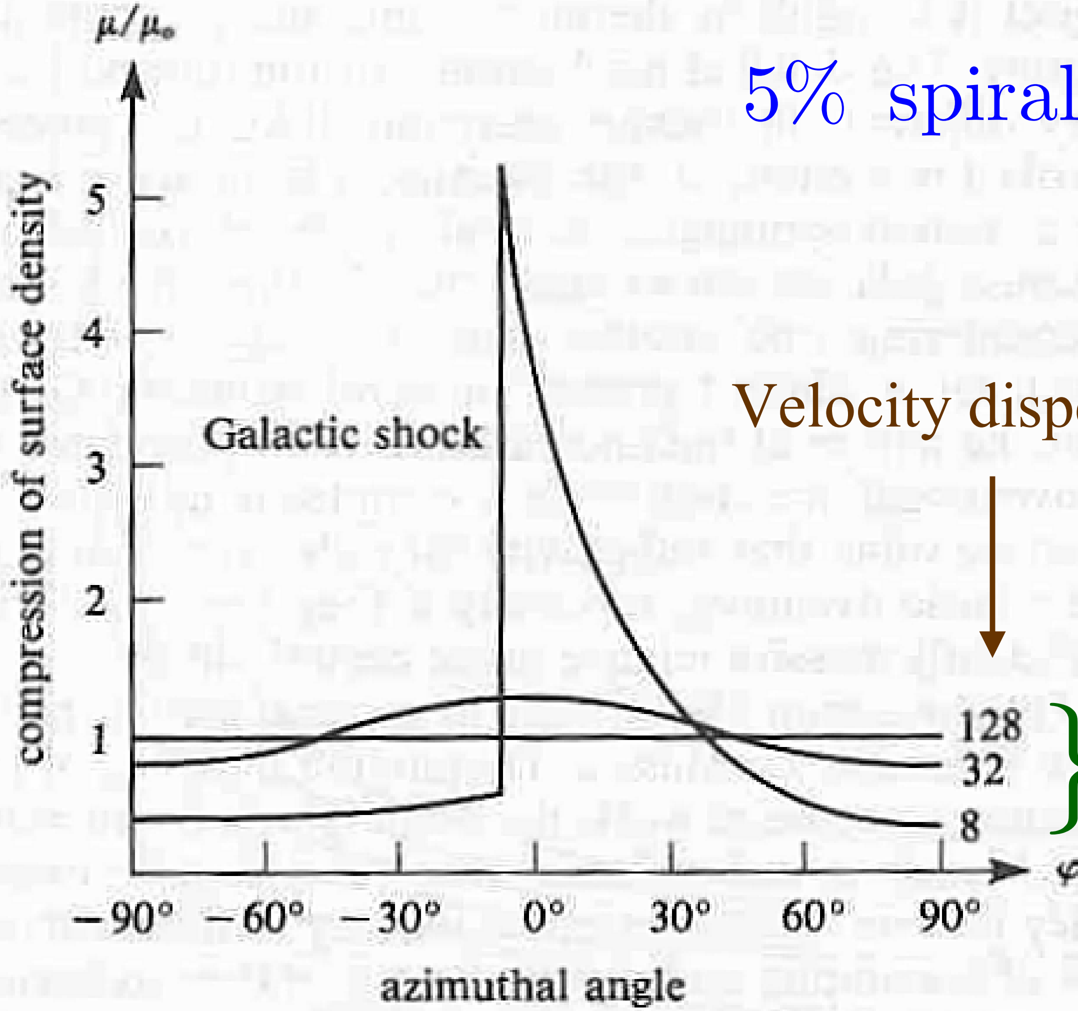
$$Q \equiv \frac{\sigma_R \kappa}{3.36\, G\Sigma} > 1$$

Stellar disk

Toomre (1964)

Cooling =>
Instability =>
Star Formation

5% spiral potential

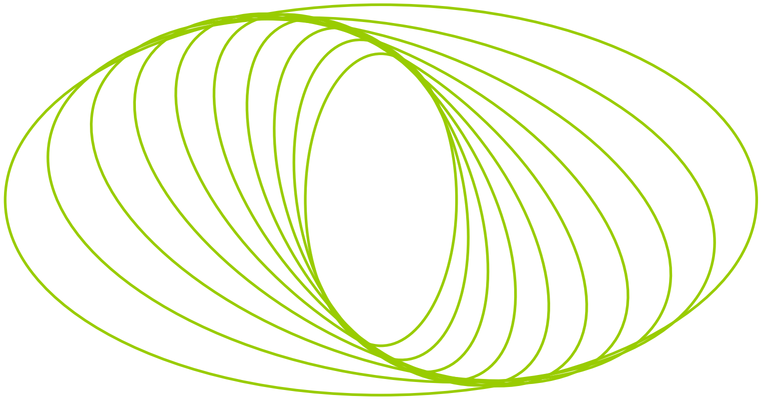


Velocity dispersion

km/s

- halo stars
- disk stars
- gas clouds

Spiral Patterns from Elliptical Stellar Orbits



<= 10 orbits

46 orbits:

NGC2997 AAT 017