Galaxies

Lecture 10

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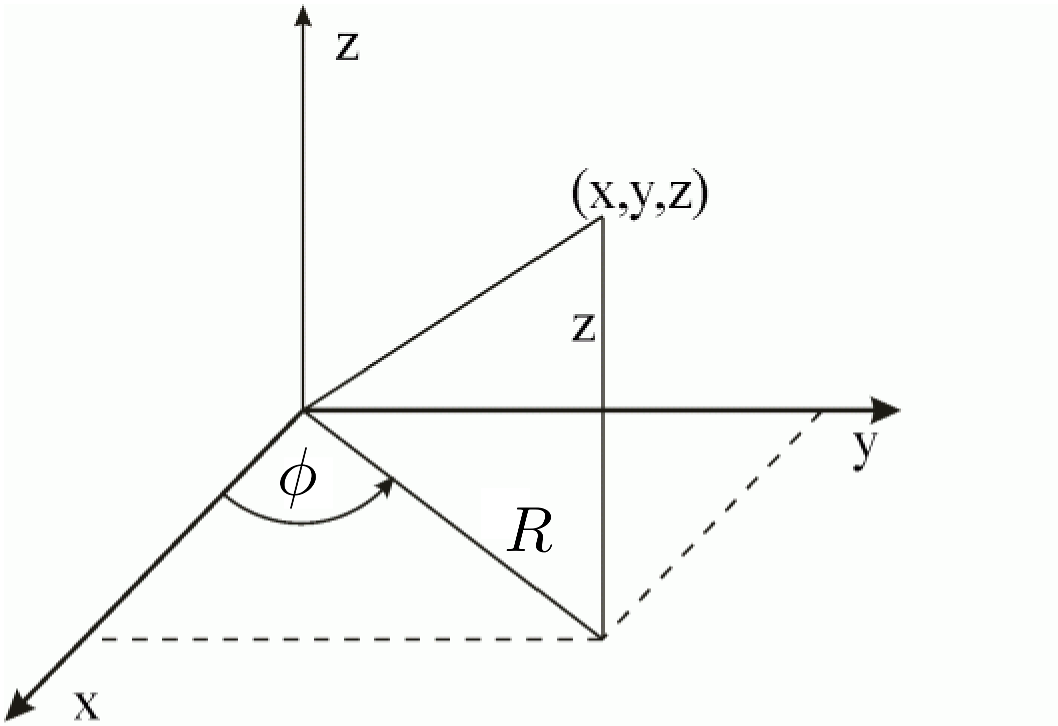
http://wise-obs.tau.ac.il/~barkana/galaxies.html

Cylindrical Coordinates: (R, ϕ, z)



$$\vec{v} = \dot{R}\,\hat{R} + R\dot{\phi}\,\hat{\phi} + \dot{z}\,\hat{z}$$

$$\frac{d\vec{v}}{dt} = (\ddot{R} - R\dot{\phi}^2)\hat{R} + (2\dot{R}\dot{\phi} + R\ddot{\phi})\hat{\phi} + \ddot{z}\,\hat{z}$$

Acceleration (Binney & Tremaine 1B-24)

NGC 891

Equation of Motion

$$\frac{d\vec{v}}{dt} = -\vec{\nabla}\Phi = -\frac{\partial\Phi}{\partial R}\hat{R} - \frac{1}{R}\frac{\partial\Phi}{\partial\phi}\hat{\phi} - \frac{\partial\Phi}{\partial z}\hat{z}$$

Axisymmetric potential: $\Phi(R, z)$

Symmetric about z: $\Phi(R, z) = \Phi(R, -z)$

$$\ddot{R} - R\dot{\phi}^2 = -\frac{\partial\Phi}{\partial R} \qquad \ddot{z} = -\frac{\partial\Phi}{\partial z} \qquad R^2\dot{\phi} = L_z = \text{const}$$

Can write as:

$$\ddot{R} = -\frac{\partial\Phi_{\text{eff}}}{\partial R} \qquad \ddot{z} = -\frac{\partial\Phi_{\text{eff}}}{\partial z} \qquad \Phi_{\text{eff}} \equiv \Phi(R, z) + \frac{L_z^2}{2R^2}$$

Circular orbit (fixed R) at a fixed z: $\vec{\nabla}\Phi_{\text{eff}} = 0$

In general, $\frac{\partial\Phi_{\text{eff}}}{\partial z} = 0$ at $z = 0$

Centripetal acceleration

Radius of circular orbit: $\left.\frac{\partial\Phi}{\partial R}\right|_{(R_g,\, z=0)} = \frac{L_z^2}{R_g^3} = R_g\dot{\phi}^2$

Theory of Epicycles: Lindblad

$$x \equiv R - R_g \qquad \Phi_{\rm eff} \simeq C + \tfrac{1}{2}\left(\kappa^2 x^2 + \nu^2 z^2\right)$$

$$\kappa^2 = \frac{\partial^2 \Phi_{\rm eff}}{\partial R^2}\Big|_{(R_g,\,0)} = \frac{\partial^2 \Phi}{\partial R^2}\Big|_{(R_g,\,0)} + \frac{3L_z^2}{R_g^4}$$

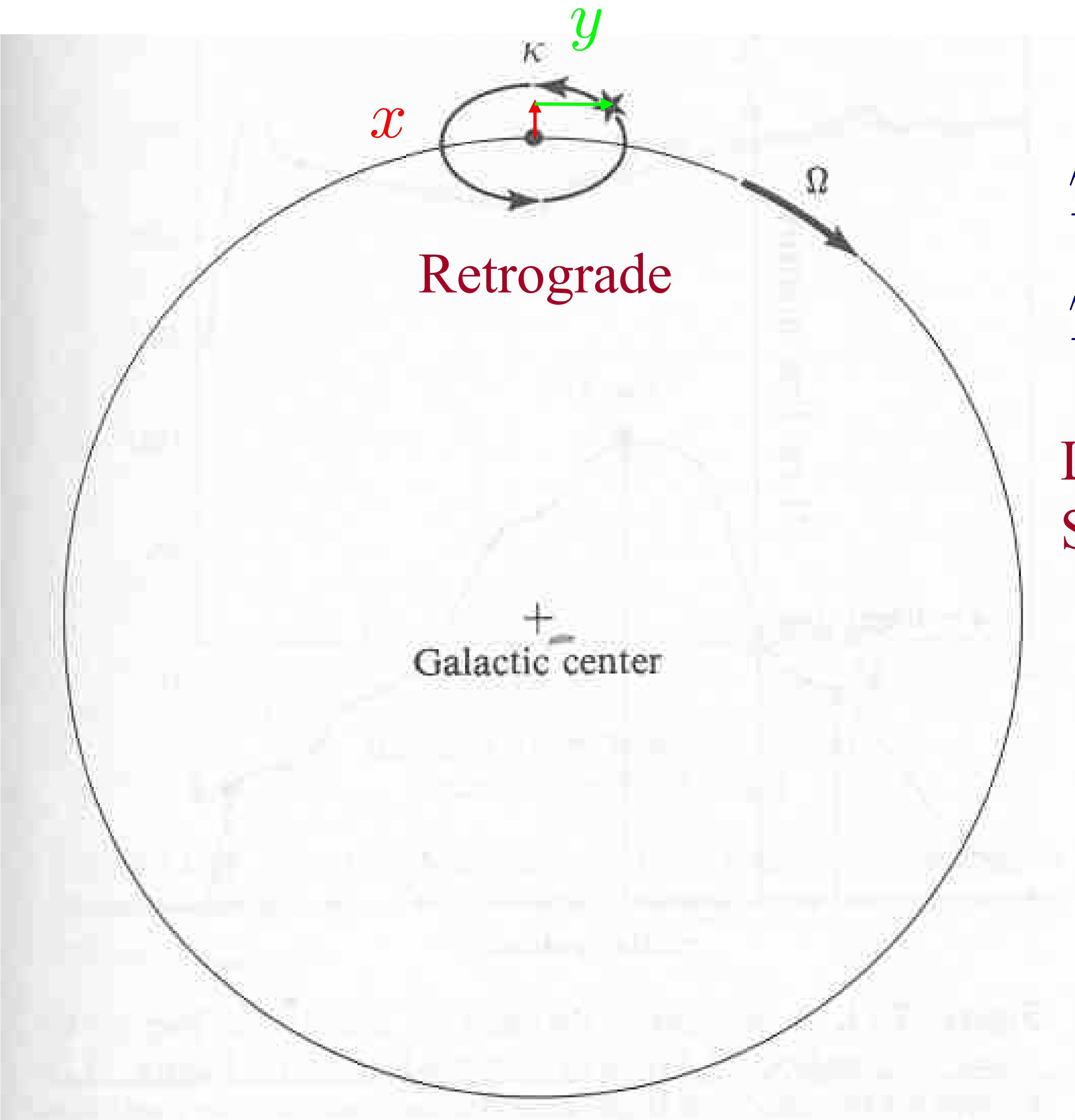
$$\nu^2 = \frac{\partial^2 \Phi}{\partial z^2}\Big|_{(R_g,\,0)} \qquad \kappa^2 = R\frac{d\Omega^2}{dR} + 4\Omega^2$$

Epicyclic frequency = κ

$$x(t) = X\cos(\kappa t + \psi_x) \qquad z(t) = Z\cos(\nu t + \psi_z)$$

Oort constants: $A = -\tfrac{1}{2}R\frac{d\Omega}{dR} \qquad B = -\left(\tfrac{1}{2}R\frac{d\Omega}{dR} + \Omega\right)$

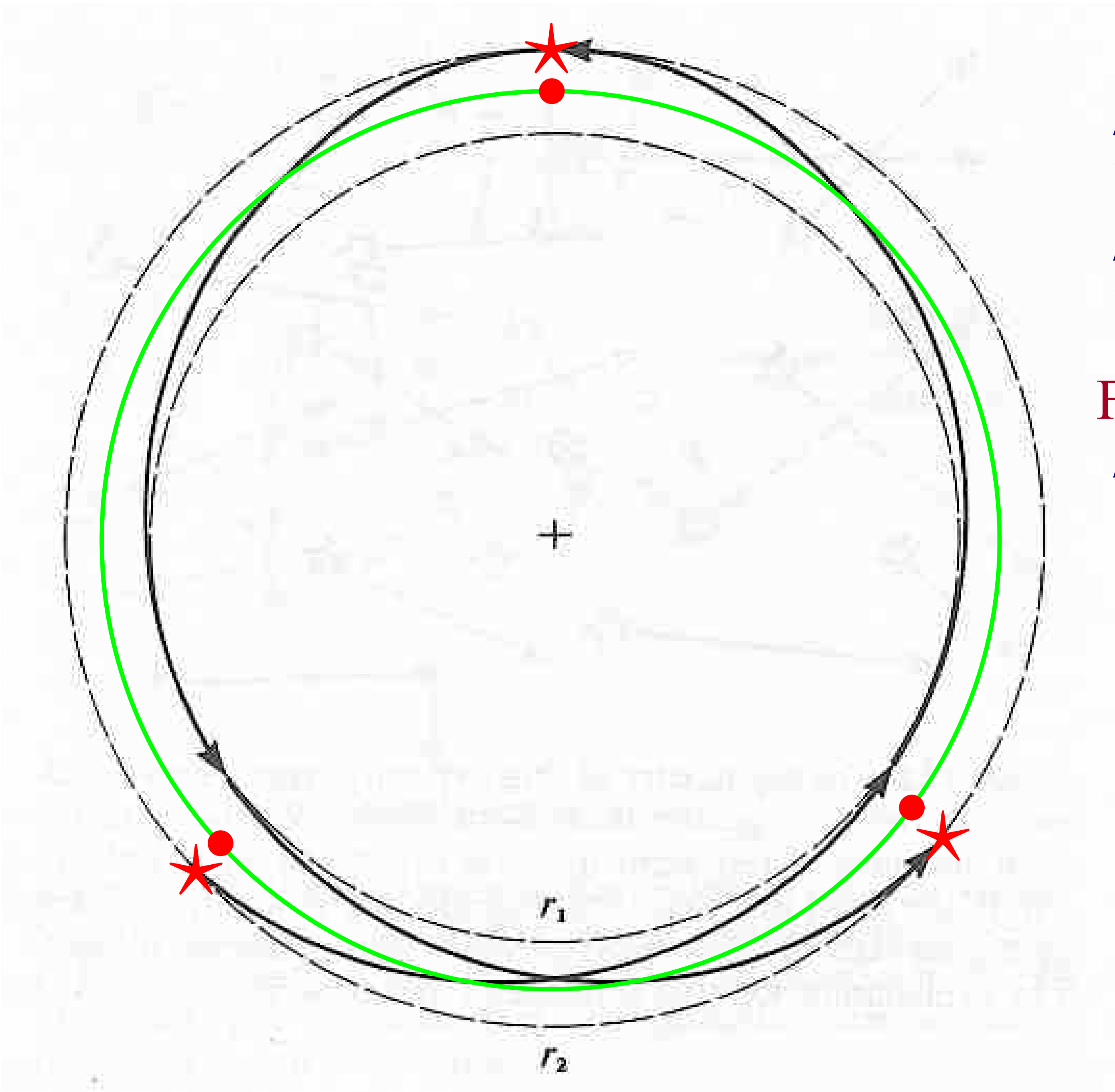
$$y(t) = -\frac{2\Omega_g}{\kappa}X\sin(\kappa t + \psi_x)$$



$$T_c = \frac{2\pi}{\Omega}$$

$$T_{\rm epi} = \frac{2\pi}{\kappa}$$

L_z conserved:
Small v at large R



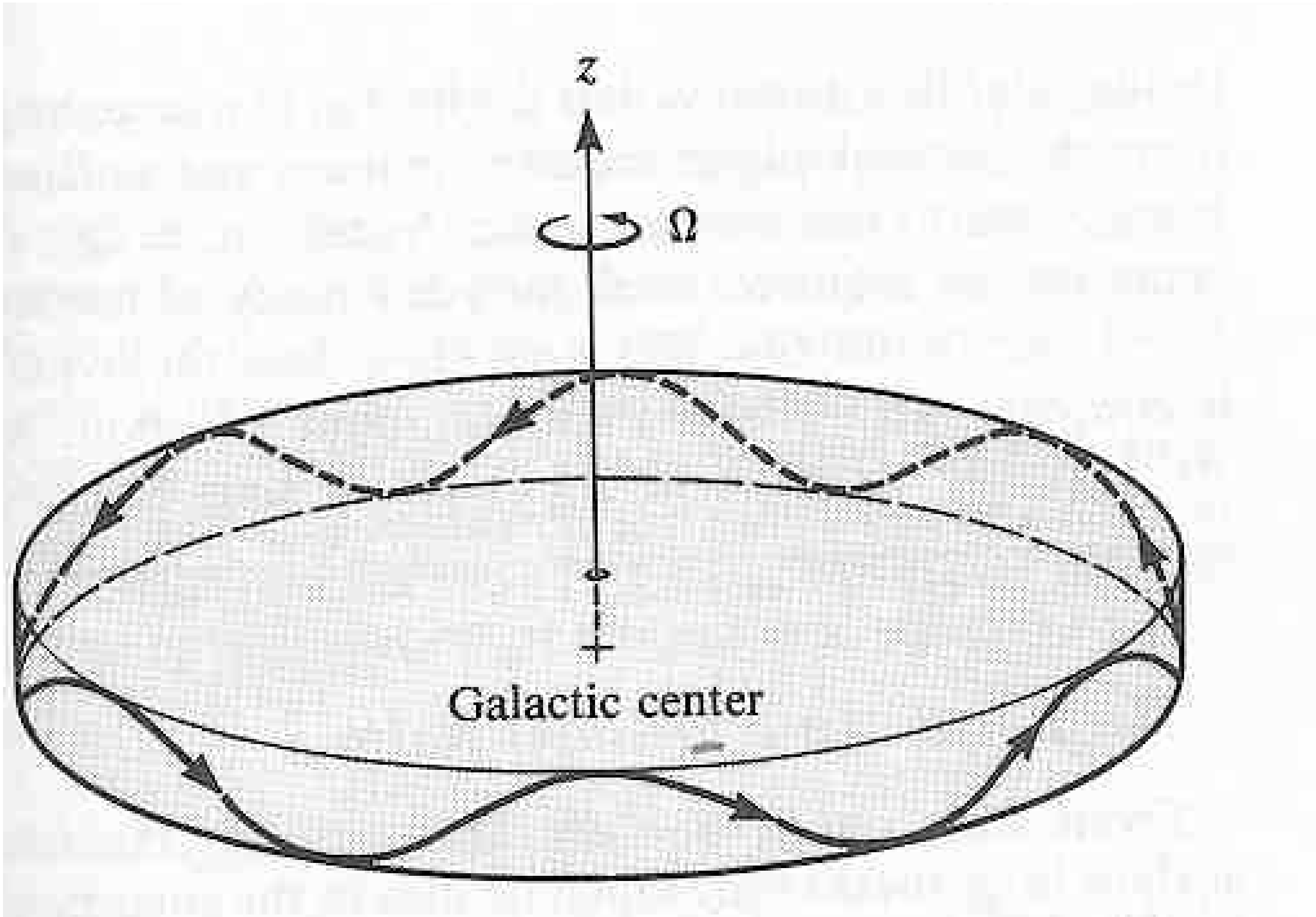
$$T_c = \frac{2\pi}{\Omega}$$

$$T_{\rm epi} = \frac{2\pi}{\kappa}$$

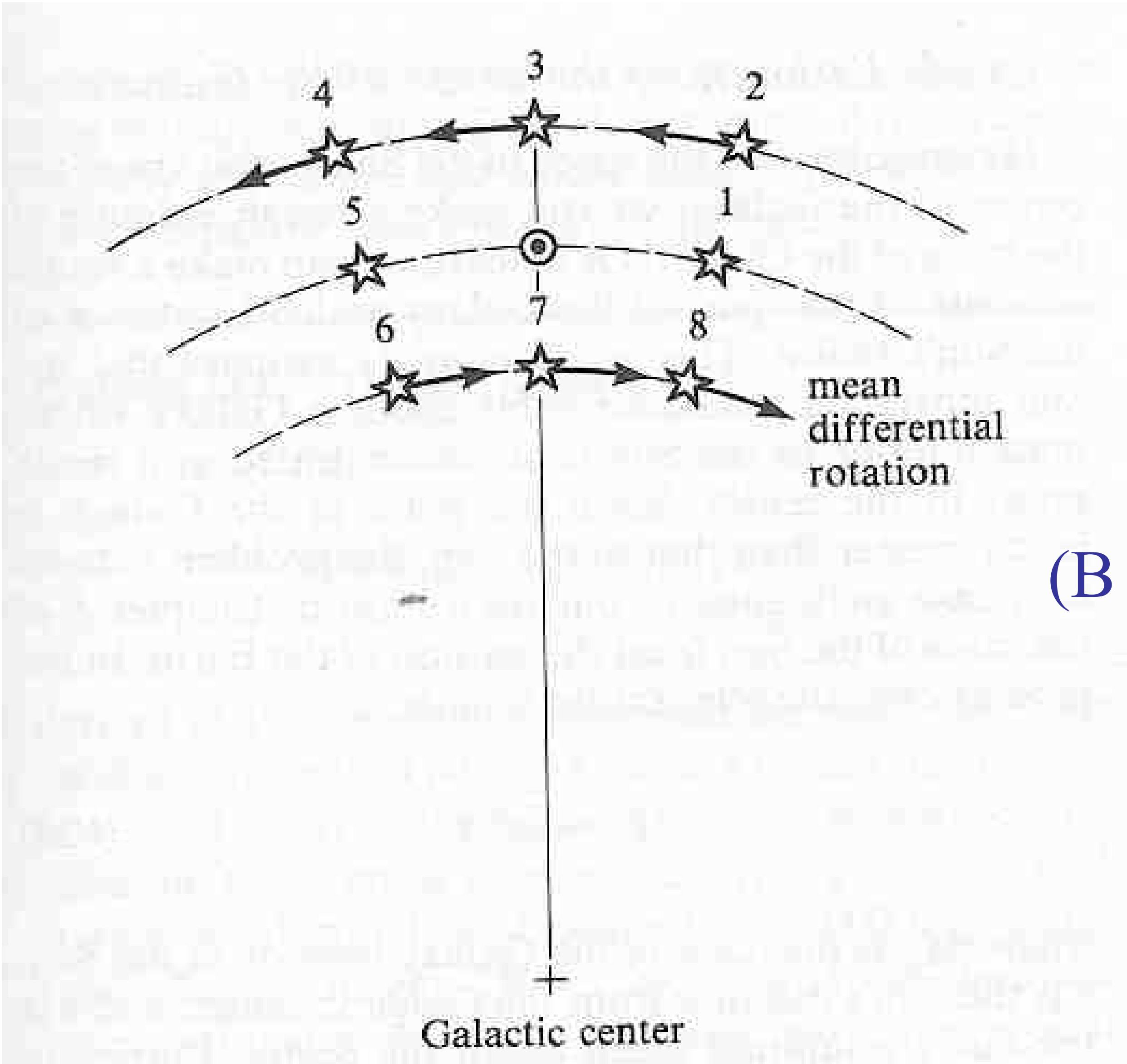
For the Sun:

$$T_{\rm epi} < T_c$$

Also: z oscillations



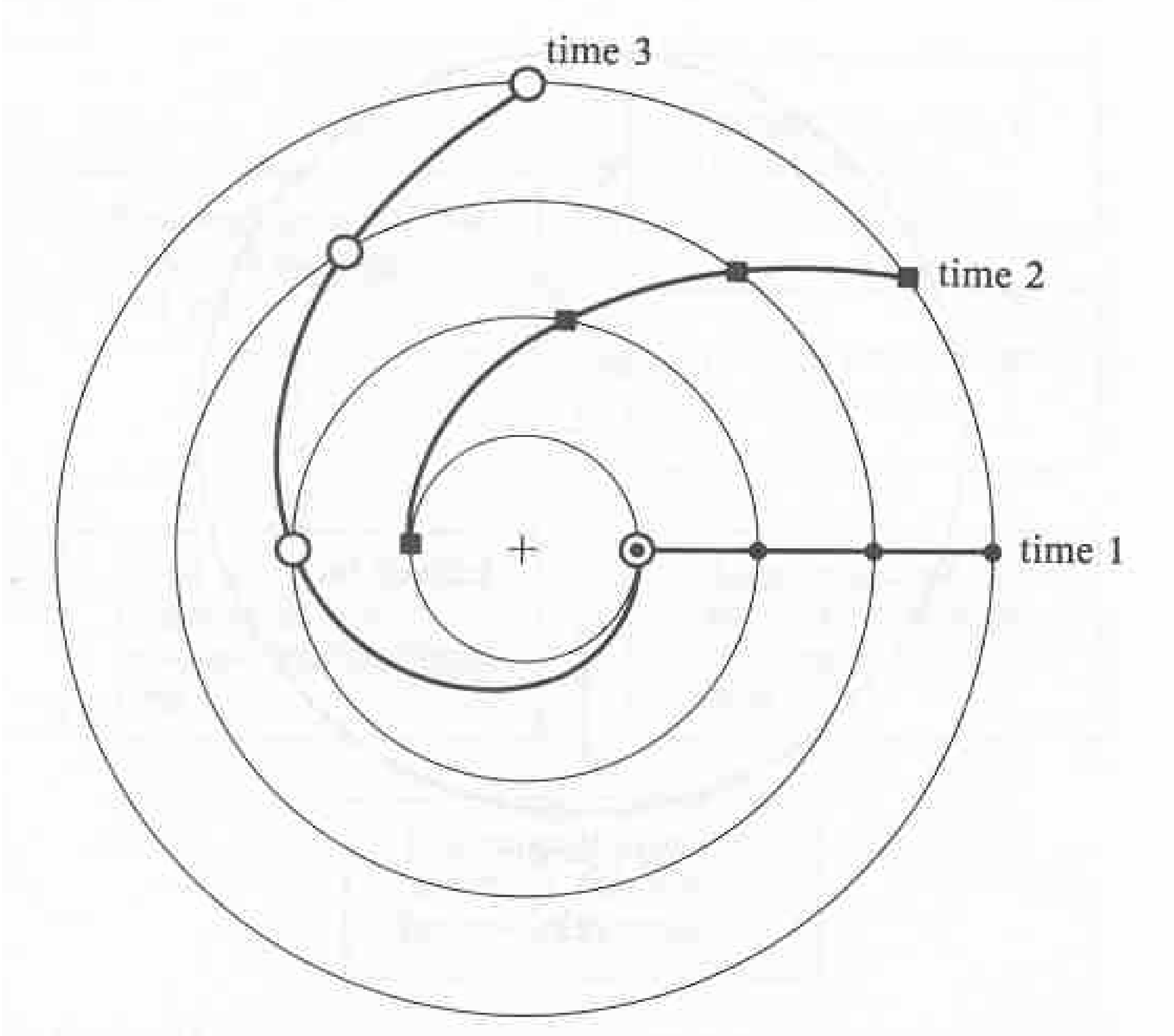
Oort constants from radial-velocity observations:



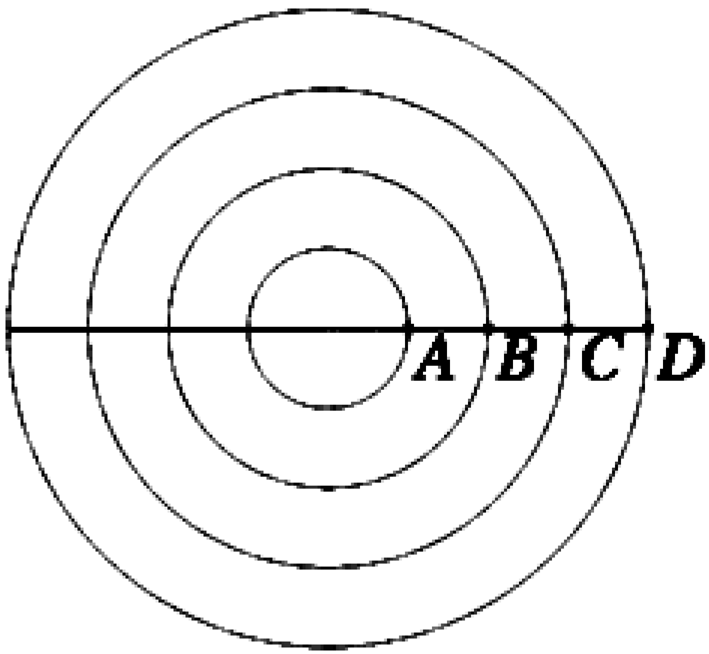
$\Rightarrow$A

(B is harder)

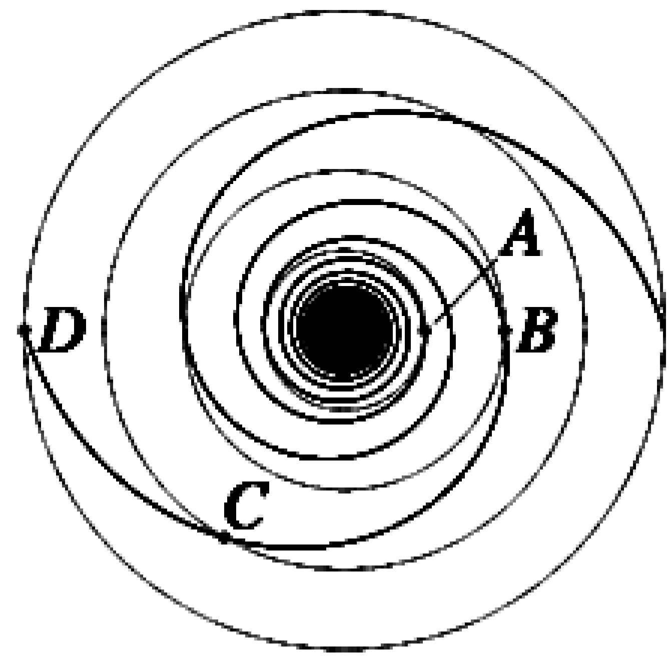
Spiral Structure: Winding Problem



Spiral Structure: Winding Problem

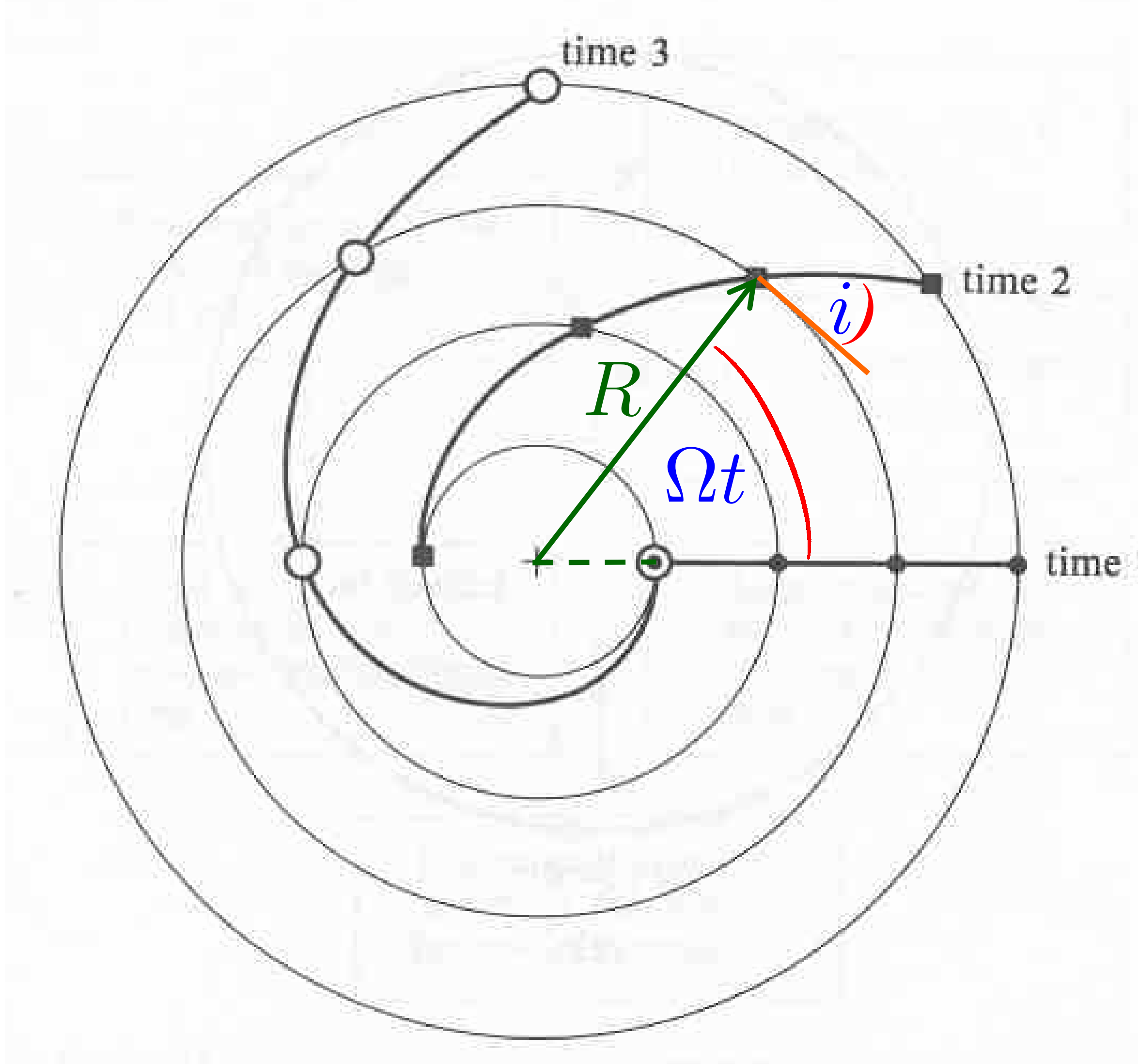


After a long time:



For the Milky Way,
expected # windings: $\sim 10^{10}\,\mathrm{yr}/(2 \times 10^{8}\,\mathrm{yr}) \simeq 50$

Spiral Structure: Winding Problem



$\phi = \phi_0$

Spiral density wave (Lin & Shu) => Potential (Poisson) => Stellar orbits

Movie:
http://burro.astr.cwru.edu/Academics/Astr222/Galaxies/Spiral/spiral.html