Cosmology 2 (Prof. Rennan Barkana): Homework 2
June 25'th, 2017; Due Thursday, July 20'th, 2017

Note: In all questions, you are allowed to use a program such as Mathematica or Matlab, and you may use numerical integration.

1. In this question on galaxy formation, assume a broken power-law as an approximate model for the standard cold dark matter power spectrum: $P(k) \propto k$ for $k < k_{\rm eq}$ and $P(k) \propto k^{-3}$ for $k > k_{\rm eq}$, where $k_{\rm eq} = 0.45\,\Omega_m h^2$ Mpc^{-1}. Set $\Omega_m = 0.307$, $\Omega_\Lambda = 1 - \Omega_m$ and $H = 67.8$ km s^{-1} Mpc^{-1} today. Note: It is best to split the integrals below into $k < k_{\rm eq}$ and $k > k_{\rm eq}$.

Calculate $\sigma(R)$, the variance of the mean density in a sphere of comoving radius R. Normalize the power spectrum so that $\sigma(R_0) = 0.829$ where $R_0 = 8h^{-1}$ Mpc, and then plot $\sigma(R)$. Also calculate and plot the correlation function $\xi(r)$, on the same plot as $\sigma(R)$ (set $R = r$). Note: In your plot, cover the range of R that is relevant to the wide range of galaxies and of large-scale structure in the universe.

Now, change the normalization of the power spectrum to some other, lower value of your choice (please do not coordinate with others in the class), and plot the same two quantities (This lower normalization corresponds to some redshift higher than zero).

[50 points]

2. In this question on spiral structure in galaxies, assume as the underlying, axisymmetric disk, a Mestel disk, which has a surface density:

$$\Sigma(R) = \frac{\Sigma_0 R_0}{R} \ .$$

a. Find $\Omega(R)$ and $\kappa(R)$ for the Mestel disk (Note: the circular velocity depends only on the disk mass enclosed within a given radius). Use this to find the radii of the Lindblad resonances for a spiral wave solution, as a function of the frequency ω and the number of arms m.

[20 points]

b. Assume $v_c = 40$ km/s, $c_S = 10$ km/s, $m = 2$, and $\omega = v_c/(10\,\text{kpc})$. Solve the dispersion relation for $k(R)$, assuming k is positive. Note: There are two solutions for k; use the one that satisfies the tight-winding approximation more accurately.

Go from $k(R)$ to the shape function, and then make a plot that shows the shape of the spiral arms in two dimensions (Hint: You may want to use a parametric plotting routine). Use different colors or line types for the different arms.

Now use one other combination of parameter values of your choice (please do not coordinate with others in the class), and again plot the shape of the spiral arms.

[30 points]