Galaxies (Dr. Rennan Barkana): Homework 2
June 15'th, 2006; Due on the date of the MOED Bet

1. In this question on galaxy formation, assume a broken power-law as an approximate model for the standard cold dark matter power spectrum: $P(k) \propto k$ for $k < k_{\rm eq}$ and $P(k) \propto k^{-3}$ for $k > k_{\rm eq}$, where $k_{\rm eq} = 0.45\, \Omega_m h^2$ Mpc^{-1}. Set $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$ and $H = 70$ km s^{-1} Mpc^{-1} today. Note: It is best to split the integrals below into $k < k_{\rm eq}$ and $k > k_{\rm eq}$.

a. Calculate $\sigma(R)$, the variance of the mean density in a sphere of comoving radius R. Normalize the power spectrum so that $\sigma(R_0) = 0.9$ where $R_0 = 8h^{-1}$ Mpc, and then plot the normalized $P(k)$ and, in a separate plot, $\sigma(R)$. Also calculate and plot the correlation function $\xi(R)$, on the same plot as $\sigma(R)$. Notes: In your plots, cover the ranges of k and R that are relevant to the wide range of galaxies and of large-scale structure in the universe. Plot log-log if this makes the plot clearer.

[25 points]

b. Using 1.a., plot the following three quantities associated with the Press-Schechter model, all versus the halo mass M: $F(> M)$, the mass fraction contained in halos of mass $> M$; dn/dM, the halo mass function; $n(> M)$, the number density of halos of mass $> M$. Note: Assume that the critical collapse overdensity is $\delta_{\rm crit} = 1.686$, although in reality it is slightly changed because of the cosmological constant.

Now, change the normalization of the power spectrum to $\sigma(R_0) = 0.165$. This corresponds approximately to the power spectrum at redshift 6. Plot the same three quantities at this redshift. Finally, change the normalization of the power spectrum to one other value of your choice, and plot the same three quantities.

[25 points]

2. In this question on spiral structure in galaxies, assume as the underlying, axisymmetric disk, a Mestel disk, which has a surface density:

$$\Sigma(R) = \frac{\Sigma_0 R_0}{R} \ ,$$

and a corresponding circular velocity which is constant with radius:

$$v_c^2 = 2\pi G \Sigma_0 R_0 \ .$$

a. Find $\Omega(R)$ and $\kappa(R)$ for the Mestel disk. Use this to find the radii of the Lindblad resonances for a spiral wave solution, as a function of the frequency ω and the number of arms m.

[20 points]

b. Assume $v_c = 40$ km/s, $c_S = 10$ km/s, $m = 2$, and $\omega = v_c/(10\,\mathrm{kpc})$. Solve the dispersion relation for $k(R)$, assuming k is positive. Note: There are two solutions for k; use the one that satisfies the tight-winding approximation more accurately. Plot $k(R)$ in the range between the Lindblad resonances.

Now make a plot that shows the shape of the spiral arms in two dimensions. Hint: It is easiest to use a parametric plotting routine in Mathematica or Matlab. Use different colors or line types for the different arms.

Now choose two other combinations of parameters, one with $m = 3$ and one with $m \geq 4$, with values of v_c, c_S, and ω that differ from those above. For each combination, plot the shape of the spiral arms.

[30 points]