Galaxies (Dr. Rennan Barkana): Homework 1
April 2'nd, 2006; Due Sunday, April 30'th, 2006

1. a. Consider a Robertson - Walker universe containing only nonrelativistic matter and a cosmological constant. Assuming the universe is flat, integrate the Friedmann equation to get explicit analytical formulas for $t(a)$ and for $a(t)$, where t is cosmic time.

[15 points]

b. With the same assumptions as in 1.a., set $\Omega_m = 0.3$ (so $\Omega_\Lambda = 0.7$) and $H = 70$ km s^{-1} Mpc^{-1} today. Using analytical formulas or numerical integrations, plot the cosmic age $t(a)$, the Hubble time $H^{-1}(a)$, and the horizon $\tau(a)$ (which is also the conformal or comoving time), all on the same plot, between $a = 10^{-3}$ and now. Find the values of these three quantities at $a = 10^{-3}$, at redshift 1, and today.

[15 points]

2. a. Prove the following statement: If we define the Fourier transform of $\delta(\vec{x})$ as:

$$\hat{\delta}(\vec{k}) = \int \frac{1}{(2\pi)^3}\, d^3x\, e^{-i\vec{k}\cdot\vec{x}}\, \delta(\vec{x}) \ ,$$

then:

$$\delta(\vec{x}) = \int d^3k\, e^{i\vec{k}\cdot\vec{x}}\, \hat{\delta}(\vec{k}) \ .$$

[15 points]

b. Consider the discrete Fourier transform defined in a cube of length L and resolution n in each dimension, as in the lecture. If we define the discrete Fourier transform of $\delta(\vec{x}_{\vec{j}})$ as:

$$\hat{\delta}(\vec{k}_{\vec{l}}) = n^{-3} \sum_{\vec{j}} e^{-i\vec{k}_{\vec{l}}\cdot\vec{x}_{\vec{j}}}\, \delta(\vec{x}_{\vec{j}}) \ ,$$

then prove that:

$$\delta(\vec{x}_{\vec{j}}) = \sum_{\vec{l}} e^{i\vec{k}_{\vec{l}}\cdot\vec{x}_{\vec{j}}}\, \hat{\delta}(\vec{k}_{\vec{l}}) \ .$$

Hint: You will get a sum which you will want to show equals zero. Show that you can multiply this sum by something different from 1 and get back the same sum. For intuition, start with the one-dimensional case and with a small number n.

[15 points]

3. a. Write down the distribution function f, as a function of the six phase-space coordinates and of time, for a uniform radiation field of Planck-distributed photons in a uniform expanding universe.

[10 points]

b. Show that this distribution function f satisfies the collisionless Boltzmann equation.

[15 points]

4. For a Gaussian probability distribution function,

$$p(x) = (2\pi\sigma^2)^{-1/2}\, e^{-x^2/(2\sigma^2)} ,$$

show that the moments are

$$\langle x^{2n} \rangle = (2n-1)!!\, \sigma^{2n} ,$$

where $(2n-1)!! = (2n-1)(2n-3)\ldots 1$. Hint: Use differentiation.

[15 points]