TEL AVIV UNIVERSITY אוניברסיטת תל-אביב

RAYMOND AND BEVERLY SACKLER FACULTY OF EXACT SCIENCES SCHOOL OF PHYSICS & ASTRONOMY

הפקולטה למדעים מדוייקים ע"ש ריימונד ובברלי סאקלר בית הספר לפיסיקה ואסטרונומיה

<u>בחינה ביצירת גלקסיות</u>

<u>סמסטר ב' תשס"ו , 2005/2006 , מועד א'</u>

<u>מרצה:</u> ד"ר רנן ברקנא **<u>תאריך:</u>** 03/07/2006

<u>משך הבחינה:</u> שלוש שעות. **<u>חומר עזר:</u>** ללא חומר עזר.

<u>הוראות:</u> ענו על השאלה הראשונה (30 נקודות), ובנוסף על שתיים מתוך שלוש השאלות האחרות (35 נקודות כל אחת). הניקוד רשום בסוף כל סעיף. שימו לב: בפיתוחים מתמטיים, הסבירו בכמה מילים כל צעד שאתם עושים, בצורה ברורה (ז"א, לא לרשום סדרה ארוכה של נוסחאות ללא הסבר).

<u>נוסחאות:</u>

$$k(R,t) = \frac{\partial f(R,t)}{\partial R} \qquad (m\Omega - \omega)^2 = \kappa^2 - 2\pi G\Sigma|k| + k^2 c_s^2$$

$$\nabla^2 \phi_P = 4\pi G\rho \qquad \kappa^2 = \frac{\partial^2 \Phi}{\partial R^2}\Big|_{z=0} + 3\Omega^2 \qquad \frac{\omega}{m} = \Omega \pm \frac{\kappa}{m}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho \vec{u}) = 0 \qquad \Sigma_1(R,\phi,t) = H(R,t)e^{i[m\phi + f(R,t)]}$$

$$\hat{\delta}(\vec{k}) \equiv \int \frac{d^3x}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{x}}\delta(\vec{x}) \qquad \sigma^2 = \int d^3k\, \tilde{W}(\vec{k})\tilde{W}(-\vec{k})P(\vec{k})$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\vec{\nabla})\vec{u} = -\vec{\nabla}\phi_{\rm P} - \frac{1}{\rho}\vec{\nabla}p - \frac{1}{\rho}\vec{\nabla}\cdot\pi$$

$$\frac{df}{dt} = \left[\frac{\partial}{\partial t} + \frac{d\vec{x}}{dt}\cdot\frac{\partial}{\partial \vec{x}} + \frac{d\vec{q}}{dt}\cdot\frac{\partial}{\partial \vec{q}}\right] f = 0$$

$$\tilde{W}_{TH}(\vec{k}) = \frac{3}{(kR)^3}\left[\sin(kR) - kR\cos(kR)\right]$$

$$\delta(\vec{x}) = \int d^3k\, e^{i\vec{k}\cdot\vec{x}}\hat{\delta}(\vec{k})$$

$$\xi(r) = 4\pi\int_0^\infty k^2 dk\, P(k)\frac{\sin(kr)}{kr}$$

Question 1: required

In this question, use words and simple equations or derivations (but not pages of full mathematical derivations).

a. What is the Jeans mass? State its meaning and estimate its size (in terms of a formula, not a numerical value) for a pure baryonic gas cloud, and for a cosmic mix of baryons and cold dark matter.
[10 points]

b. What (roughly) is the shape of the density power spectrum in the universe at present? Why does it have that shape? (Explain in general terms, not in full detail.) Where is its peak? (Give a rough number.) How does the peak position depend on cosmological parameters?
[10 points]

c. In class we derived an equation for a spherical stellar system:

$$\frac{1}{n}\frac{d}{dR}\left(n\,\overline{v_R^2}\right) + 2\beta\,\frac{\overline{v_R^2}}{R} = -\frac{d\Phi}{dR}$$

Explain in general terms how this equation is derived, and also how it is applied in astrophysics.
[10 points]

Question 2

Consider spiral structure within a hypothetical gas disk where the mass is strongly dominated by a central black hole of mass M_b . You may assume the mass of the gas in the disk is negligible compared with M_b . You may also assume the tight-winding approximation. The gas in the disk has a sound speed c_S which you may assume is large (i.e., comparable to the rotation speed).

a. Find $\Omega(R)$, $\kappa(R)$, and the potential $\Phi(R)$ for this disk, where R is the two-dimensional radius.
[10 points]

b. Find the radii of the Lindblad resonances and of the corotation resonance, for a given wave solution with temporal frequency ω and m spiral arms.
[10 points]

c. Find the dispersion relation for this disk, at $R>0$, assuming $\omega \ll \Omega(R)$. Find an expression in this case for the total number of windings (i.e., full circles) of a spiral arm as it goes from some radius R_1 to some larger radius R_2 .
[10 points]

d. Analyze the stability of this disk to axisymmetric perturbations. (Do not assume $\omega \ll \Omega$.)
[5 points]

Question 3

a. Suppose we wish to average a field of density fluctuations δ over a window function which is a linear segment of length L in some fixed direction. Write down the window function and find its Fourier transform. (Hint: It's best to define a symmetric window function with respect to the coordinate origin.) Then find an expression for the root mean square fluctuation σ_L of the average density on the linear segment.
[15 points]

b. Assume a power-law power spectrum, $P(k) \propto k^n$ and a density field that grows with time in proportion to the growing mode in an Einstein de Sitter universe. Calculate how the r.m.s. fluctuation σ in spheres of radius R scales with R and with redshift (Hint: Use the variable x=kR). Calculate how σ_L (which you found in part a) scales with L and with redshift. Calculate how the typical mass of collapsing halos scales with redshift. Calculate how the typical virial temperature of collapsing halos scales with redshift. (The virial temperature is the temperature to which gas is heated when it virializes within a dark matter halo. You may use results that we derived in class about virialization.)
[20 points]

Question 4

In this question, a dot means a derivative with respect to cosmic time t.

a. In class we showed that linear perturbations outside the horizon evolve according to: $-\frac{2H}{n}\dot{\delta} = H^2\delta - \frac{k}{a^2}$

where we assume a single density component with $\rho \propto a^{-n}$, and H and a refer to the unperturbed, flat universe.

Assuming a power-law solution, calculate how δ scales with a. Also show how δ scales with cosmic time t, and how δ scales with conformal time τ . In addition, find all these scalings for the particular cases of a matter-dominated universe, and a radiation-dominated universe.

[15 points]

b. In class we showed that linear Newtonian perturbations evolve according to: $\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\rho\delta$

where ρ and a refer to the unperturbed, flat universe. Solve this equation for the case of a single density component with $\rho \propto a^{-n}$.

[10 points]

c. Now generalize part b to the case where again there is a single density component with $\rho \propto a^{-n}$, but only a fraction f of it is perturbed, and the rest is always spatially homogeneous. Solve the equation for δ in general, and then, for the case of cold dark matter, find the solutions in the specific cases f=0, f=1/3, and f=1.

[10 points]